Seesaw Invariance of Fritzsch-like Neutrino Mass Matrices, Leptogenesis and Lepton Flavor Violation

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Abstract

We propose a simple ansatz for neutrino phenomenology, in which the relevant lepton mass matrices take the universal Fritzsch-like form and the seesaw relation holds under a particular condition. There exist six textures of this nature, but their consequences on neutrino oscillations are exactly the same. We show that our scenario is viable to account for the cosmological baryon number asymmetry via thermal leptogenesis. Its predictions for the lepton-flavor-violating decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ are also presented. We find that the branching ratios of these rare processes depend strongly upon the phase parameters responsible for leptogenesis and for leptonic CP violation in neutrino oscillations.

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I. INTRODUCTION

The Super-Kamiokande [1], SNO [2], KamLAND [3] and K2K [4] neutrino oscillation experiments have provided us with very compelling evidence that neutrinos are massive and lepton flavors are mixed. This important result indicates that the minimal supersymmetric standard model (MSSM) is actually as incomplete as the standard model in describing the neutrino phenomenology. A very simple but natural extension of the MSSM is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge transformation. In this case, the Lagrangian responsible for lepton masses can be written as

$$-\mathcal{L}_{\rm lepton} \; = \; \bar{l}_{\rm L} Y_l E H_1 + \bar{l}_{\rm L} Y_\nu N H_2 + \frac{1}{2} \overline{N^{\rm c}} M_{\rm R} N + {\rm h.c.} \; , \tag{1}$$

where $l_{\rm L}$ denotes the left-handed lepton doublet, E and N stand respectively for the right-handed charged lepton and Majorana neutrino singlets, and H_1 and H_2 (with hypercharges $\pm 1/2$) are the MSSM Higgs doublets. After spontaneous gauge symmetry breaking, one obtains the charged lepton mass matrix $M_l \equiv Y_l \langle H_1 \rangle$ and the Dirac neutrino mass matrix $M_{\rm D} \equiv Y_\nu \langle H_2 \rangle$. The scale of $M_{\rm R}$ may be considerably higher than $\langle H_{1,2} \rangle$, because right-handed neutrinos are $\mathrm{SU(2)_L}$ singlets and their mass term is not subject to electroweak symmetry breaking. Thus the effective neutrino mass matrix M_ν can be derived from $M_{\rm D}$ and $M_{\rm R}$ via the seesaw relation $M_\nu = M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ [5]. Although this elegant relation can qualitatively attribute the smallness of left-handed neutrino masses to the largeness of right-handed neutrino masses, it is unable to make any concrete predictions unless a specific lepton flavor structure is assumed. Hence an appropriate combination of the seesaw mechanism and possible flavor symmetries or texture zeros [6] is practically needed, in order to quantitatively account for the neutrino mass spectrum and lepton flavor mixing.

One purpose of this paper is to incorporate the seesaw mechanism with the Fritzsch-like textures of lepton mass matrices listed in Table 1. Those six patterns of M_l and M_ν are actually isomeric [7]; i.e., they have the same phenomenological consequences, although their structures are apparently different from one another. If $M_{\rm D}$ and $M_{\rm R}$ take the same Fritzsch-like form as M_l and M_ν do, then the seesaw relation $M_\nu = M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ will in general be violated. We shall show that this relation can keep unchanged, provided the condition $\mathcal{B}_{\rm D}/\mathcal{C}_{\rm D} = \mathcal{B}_{\rm R}/\mathcal{C}_{\rm R}$ is satisfied. In this case, each nonvanishing matrix element of M_ν has a simple seesaw relation with its counterparts in $M_{\rm D}$ and $M_{\rm R}$; i.e.,

$$\mathcal{A}_\nu = \frac{\mathcal{A}^2_{\rm D}}{\mathcal{A}_{\rm R}} \; , \quad \mathcal{B}_\nu = \frac{\mathcal{B}^2_{\rm D}}{\mathcal{B}_{\rm R}} \; , \quad \mathcal{C}_\nu = \frac{\mathcal{C}^2_{\rm D}}{\mathcal{C}_{\rm R}} \; . \tag{2}$$

The textures of $M_{\rm D}$, $M_{\rm R}$ and M_ν in Table 1 are therefore referred to as the seesaw-invariant textures. Because the parameters of M_ν can essentially be determined from current neutrino oscillation data [8], it is then possible to impose useful constraints on the parameters of $M_{\rm D}$ and $M_{\rm R}$ via Eq. (2).

Another purpose of this paper is to account for the cosmological baryon number asymmetry via thermal leptogenesis [9]. Indeed, lepton number violation induced by the third term of $\mathcal{L}_{\rm lepton}$ allows decays of the heavy Majorana neutrinos N_i (for $i = 1, 2, 3$) to happen.

Since the decay can occur at both tree and one-loop levels, their interference may result in a CP-violating asymmetry ε_i between the CP-conjugated $N_i \rightarrow l + H_2^c$ and $N_i \rightarrow l^c + H_2$ processes. If the masses of N_i are hierarchical (i.e., $M_1 < M_2 < M_3$), the interactions of N_1 can be in thermal equilibrium when N_2 and N_3 decay. The asymmetries ε_2 and ε_3 are therefore erased before N_1 decays, and only the asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives. The point of leptogenesis [9] is that ε_1 may give rise to a net lepton number asymmetry $Y_{\rm L} \equiv (n_{\rm L} - n_{\bar{\rm L}})/\mathbf{s} \propto \varepsilon_1$, which is eventually converted into a net baryon number asymmetry $Y_{\rm B}$ via nonperturbative sphaleron processes [10]: $Y_{\rm B} \equiv (n_{\rm B} - n_{\bar{\rm B}})/\mathbf{s} \propto Y_{\rm L}$. Thus this mechanism provides a natural interpretation of the cosmological matter-antimatter asymmetry, $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$, which is drawn from the recent WMAP observational data [11]. We shall show that six Fritzsch-like textures of lepton mass matrices in Table 1 yield the same ε_1 and $Y_{\rm B}$, from which useful constraints on the mass scale of three right-handed neutrinos and the Majorana phase of CP violation can be obtained.

The third purpose of this paper is to calculate the rare lepton-flavor-violating processes $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$. We shall focus our attention on a rather conservative case of lepton flavor violation, based on the scenarios where supersymmetry is broken in a hidden sector and the breaking is transmitted to the observable sector by a flavor-blind mechanism. We find that the values of ${\rm Br}(l_j \rightarrow l_i\gamma)$ depend strongly upon the phase parameters responsible for leptogenesis and for CP violation in neutrino oscillations.

II. SEESAW-INVARIANT TEXTURES

Now let us show that the seesaw relation $M_\nu = M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ will hold for each of the six patterns listed in Table 1, if and only if the condition $\mathcal{B}_{\rm D}/\mathcal{C}_{\rm D} = \mathcal{B}_{\rm R}/\mathcal{C}_{\rm R}$ is satisfied. To be explicit, we take pattern (A) – namely, the Fritzsch texture [12], for example. Given the Fritzsch form of $M_{\rm D}$ and $M_{\rm R}$, the seesaw relation leads straightforwardly to

$$M_\nu = \begin{pmatrix} \mathbf{0} & \dfrac{\mathcal{C}_{\rm D}^2}{\mathcal{C}_{\rm R}} & \mathbf{0} \\ \dfrac{\mathcal{C}_{\rm D}^2}{\mathcal{C}_{\rm R}} & \mathbf{0} & \dfrac{\mathcal{B}_{\rm D}\mathcal{C}_{\rm D}}{\mathcal{C}_{\rm R}} \\ \mathbf{0} & \dfrac{\mathcal{B}_{\rm D}\mathcal{C}_{\rm D}}{\mathcal{C}_{\rm R}} & \dfrac{\mathcal{A}_{\rm D}^2}{\mathcal{A}_{\rm R}} \end{pmatrix} + \frac{\mathcal{C}_{\rm D}^2}{\mathcal{A}_{\rm R}} \left(\frac{\mathcal{B}_{\rm D}}{\mathcal{C}_{\rm D}} - \frac{\mathcal{B}_{\rm R}}{\mathcal{C}_{\rm R}} \right) \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dfrac{\mathcal{B}_{\rm D}}{\mathcal{C}_{\rm D}} - \dfrac{\mathcal{B}_{\rm R}}{\mathcal{C}_{\rm R}} & \dfrac{\mathcal{A}_{\rm D}}{\mathcal{C}_{\rm D}} \\ \mathbf{0} & \dfrac{\mathcal{A}_{\rm D}}{\mathcal{C}_{\rm D}} & \mathbf{0} \end{pmatrix} . \tag{3}$$

One can see that the second term of Eq. (3) will vanish, if $\mathcal{B}_{\rm D}/\mathcal{C}_{\rm D} = \mathcal{B}_{\rm R}/\mathcal{C}_{\rm R}$ is taken. In this case, the (2,3) or (3,2) element of M_ν turns out to be $\mathcal{B}_{\rm D}\mathcal{C}_{\rm D}/\mathcal{C}_{\rm R} = \mathcal{B}_{\rm D}^2/\mathcal{B}_{\rm R}$. Then Eq. (3) is simplified to

$$M_\nu = \begin{pmatrix} \mathbf{0} & \dfrac{\mathcal{C}_{\rm D}^2}{\mathcal{C}_{\rm R}} & \mathbf{0} \\ \dfrac{\mathcal{C}_{\rm D}^2}{\mathcal{C}_{\rm R}} & \mathbf{0} & \dfrac{\mathcal{B}_{\rm D}^2}{\mathcal{B}_{\rm R}} \\ \mathbf{0} & \dfrac{\mathcal{B}_{\rm D}^2}{\mathcal{B}_{\rm R}} & \dfrac{\mathcal{A}_{\rm D}^2}{\mathcal{A}_{\rm R}} \end{pmatrix} . \tag{4}$$

Comparing between the form of M_ν in Eq. (4) and that defined in Table 1, we immediately arrive at the elegant seesaw relations given in Eq. (2). This result implies that

$$\frac{\mathcal{B}_{\rm D}}{\mathcal{C}_{\rm D}} \; = \; \frac{\mathcal{B}_{\rm R}}{\mathcal{C}_{\rm R}} \; = \; \frac{\mathcal{B}_\nu}{\mathcal{C}_\nu} \tag{5}$$

holds. Eq. (5) imposes a strong constraint on the structure of $M_{\rm D}$, $M_{\rm R}$ and M_ν. Because the magnitudes of $\mathcal{A}_\nu$, $\mathcal{B}_\nu$ and $\mathcal{C}_\nu$ only have a quite weak hierarchy as required by current experimental data [7], we are not allowed to assume $\mathcal{B}_\nu/\mathcal{C}_\nu = \mathcal{B}_l/\mathcal{C}_l$ in a similar way. Indeed, $|\mathcal{B}_\nu|/|\mathcal{C}_\nu| \ll |\mathcal{B}_l|/|\mathcal{C}_l|$ must hold. Without loss of generality, we take $\mathcal{A}_l$, $\mathcal{A}_\nu$, $\mathcal{A}_{\rm D}$ and $\mathcal{A}_{\rm R}$ to be real and positive. Then only the off-diagonal elements of M_a (for $a = l$, D, R or ν) are complex. It is possible to decompose M_a into $M_a = P_a \overline{M}_a P^T_a$, where

$$\overline{M}_a \; = \; \left(\begin{matrix} \mathbf{0} & C_a & \mathbf{0} \\ C_a & \mathbf{0} & B_a \\ \mathbf{0} & B_a & A_a \end{matrix} \right) \tag{6}$$

and $P_a = {\rm Diag}\{e^{i(\varphi_a - \phi_a)}, \; e^{i\phi_a}, \; 1\}$ with $A_a = \mathcal{A}_a$, $B_a = |\mathcal{B}_a|$, $C_a = |\mathcal{C}_a|$, $\phi_a \equiv \arg(\mathcal{B}_a)$ and $\varphi_a \equiv \arg(\mathcal{C}_a)$. Eq. (2) indicates that $\phi_\nu = 2\phi_{\rm D} - \phi_{\rm R}$ and $\varphi_\nu = 2\varphi_{\rm D} - \varphi_{\rm R}$ hold. Hence one can get the phase relation $\phi_{\rm D} - \varphi_{\rm D} = \phi_{\rm R} - \varphi_{\rm R} = \phi_\nu - \varphi_\nu$ from Eq. (5). Because of ${\rm Det}(\overline{M}_a) = -A_a C^2_a < 0$, it is more convenient to diagonalize $\overline{M}_a$ by using the transformation

$$(O_a Q)^T \, \overline{M}_a \, (O_a Q) \; = \; \left(\begin{matrix} \lambda^a_1 & 0 & 0 \\ 0 & \lambda^a_2 & 0 \\ 0 & 0 & \lambda^a_3 \end{matrix} \right) \; , \tag{7}$$

where O_a denotes a *real* orthogonal matrix, $Q = {\rm Diag}\{1, i, 1\}$ is a pure phase matrix defined to cancel the minus sign of ${\rm Det}(\overline{M}_a)$, and λ^a_i (for $i = 1, 2, 3$) stand for the *positive* eigenvalues of $\overline{M}_a$. Then we have

$$\begin{aligned} A_a &= \lambda^a_1 - \lambda^a_2 + \lambda^a_3 \; , \\ B_a &= \left[\frac{(\lambda^a_1 - \lambda^a_2)\,(\lambda^a_2 - \lambda^a_3)\,(\lambda^a_1 + \lambda^a_3)}{\lambda^a_1 - \lambda^a_2 + \lambda^a_3} \right]^{1/2} \; , \\ C_a &= \left(\frac{\lambda^a_1 \lambda^a_2 \lambda^a_3}{\lambda^a_1 - \lambda^a_2 + \lambda^a_3} \right)^{1/2} \; . \end{aligned} \tag{8}$$

Defining the dimensionless ratios $x_a \equiv \lambda^a_1/\lambda^a_2$ and $z_a \equiv \lambda^a_1/\lambda^a_3$, we further obtain

$$\begin{aligned} O^a_{11} &= + \left[\frac{x_a - z_a}{(1 + x_a)\,(1 - z_a)\,(x_a - z_a + x_a z_a)} \right]^{1/2} \; , \\ O^a_{12} &= - \left[\frac{x^3_a\,(1 + z_a)}{(1 + x_a)\,(x_a + z_a)\,(x_a - z_a + x_a z_a)} \right]^{1/2} \; , \\ O^a_{13} &= + \left[\frac{z^3_a\,(1 - x_a)}{(1 - z_a)\,(x_a + z_a)\,(x_a - z_a + x_a z_a)} \right]^{1/2} \; , \end{aligned}$$

$$
\begin{aligned}
O^a_{21} &= +\left[\frac{x_a - z_a}{(1+x_a)(1-z_a)}\right]^{1/2} , \\
O^a_{22} &= +\left[\frac{x_a(1+z_a)}{(1+x_a)(x_a+z_a)}\right]^{1/2} , \\
O^a_{23} &= +\left[\frac{z_a(1-x_a)}{(1-z_a)(x_a+z_a)}\right]^{1/2} , \\
O^a_{31} &= -\left[\frac{x_a z_a(1-x_a)(1+z_a)}{(1+x_a)(1-z_a)(x_a-z_a+x_a z_a)}\right]^{1/2} , \\
O^a_{32} &= -\left[\frac{z_a(1-x_a)(x_a-z_a)}{(1+x_a)(x_a+z_a)(x_a-z_a+x_a z_a)}\right]^{1/2} , \\
O^a_{33} &= +\left[\frac{x_a(1+z_a)(x_a-z_a)}{(1-z_a)(x_a+z_a)(x_a-z_a+x_a z_a)}\right]^{1/2} .
\end{aligned}
\tag{9}
$$

The full calculability of M_a is quite encouraging, because it implies that the parameters of $M_{\rm D}$ can be determined in terms of those of M_ν and $M_{\rm R}$:

$$
A_{\rm D} = \sqrt{A_\nu A_{\rm R}} , \quad B_{\rm D} = \sqrt{B_\nu B_{\rm R}} , \quad C_{\rm D} = \sqrt{C_\nu C_{\rm R}} . \tag{10}
$$

It is then possible to link the observables of leptogenesis and lepton flavor violation to the masses of three light neutrinos (m_i) and three heavy neutrinos (M_i) in a rather simple way.

One may straightforwardly show that the other five patterns of neutrino mass matrices in Table 1 are also seesaw-invariant under the condition $\mathcal{B}_{\rm D}/\mathcal{C}_{\rm D} = \mathcal{B}_{\rm R}/\mathcal{C}_{\rm R}$. Thus Eqs. (2), (5) and (10) are universally valid. We diagonalize M_a via the transformation

$$
(P^*_a O_a Q)^T M_a (P^*_a O_a Q) \; = \; (O_a Q)^T \overline{M}_a (O_a Q) \; , \tag{11}
$$

as defined in Eq. (7). The relevant forms of P_a and O_a are listed in Table 1 for six Fritzsch-like textures of lepton mass matrices. Then we find that Eqs. (8) and (9) universally hold. This result allows us to discuss six isomeric patterns of M_a in a uniform way.

The phenomenon of lepton flavor mixing arises from the mismatch between diagonalizations of M_l and M_ν. It is described by the unitary matrix $V = (P^*_l O_l Q)^T (P^*_\nu O_\nu Q)^*$. Explicitly,

$$
|V_{pq}| \; = \; \left| O^l_{1p} O^\nu_{1q} e^{i\alpha} \; + \; O^l_{2p} O^\nu_{2q} e^{i\beta} \; + \; O^l_{3p} O^\nu_{3q} \right| \; , \tag{12}
$$

where the subscripts p and q run respectively over (e, μ, τ) and $(1, 2, 3)$, and the phases α and β are defined by $\alpha \equiv (\varphi_\nu - \varphi_l) - \beta$ and $\beta \equiv (\phi_\nu - \phi_l)$. It is clear that $|V_{pq}|$ depend only upon four free parameters: x_ν, z_ν, α and β, because the mass ratios of charged leptons x_l and z_l are well known. The parameter space of (x_ν, z_ν) and (α, β) can be determined from current experimental data on solar, atmospheric and reactor neutrino oscillations. A detailed analysis has been done in Ref. [7]. It is found that the Fritzsch-like textures of M_l and M_ν can fit the present data at the 3σ level.

Instead of repeating the analysis done before, we shall subsequently concentrate on the consequences of our phenomenological ansatz on thermal leptogenesis and lepton flavor violation.

III. THERMAL LEPTOGENESIS

As argued above, we assume that three heavy right-handed neutrinos have a clear mass hierarchy and the out-of-equilibrium decay of the lightest one (N_1) is the only source of lepton number asymmetry. In the flavor basis where both M_l and $M_{\rm R}$ are diagonal, real and positive, the CP-violating asymmetry between $N_1 \to l + H_2^{\rm c}$ and $N_1 \to l^{\rm c} + H_2$ processes reads [13]

$$\begin{aligned}\varepsilon_1 &\equiv \frac{\Gamma(N_1 \to l + H_2^{\rm c}) - \Gamma(N_1 \to l^{\rm c} + H_2)}{\Gamma(N_1 \to l + H_2^{\rm c}) + \Gamma(N_1 \to l^{\rm c} + H_2)} \\ &\approx -\frac{3}{8\pi} \cdot \frac{x_{\rm R} {\rm Im}\left[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{12}\right]^2 + z_{\rm R} {\rm Im}\left[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{13}\right]^2}{\langle H_2\rangle^2 (\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{11}} \ , \end{aligned} \tag{13}$$

in which $x_{\rm R} \equiv M_1/M_2$ and $z_{\rm R} \equiv M_1/M_3$ with a normal mass hierarchy $z_{\rm R}^2 \ll x_{\rm R}^2 \ll 1$, $\langle H_2\rangle = v \sin\beta_{\rm susy}$ with $v \approx 174$ GeV, and $\tilde{M}_{\rm D} = (P_l^* O_l Q)^T M_{\rm D} (P_{\rm R}^* O_{\rm R} Q)$. Note that $\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D}$ can be expressed as

$$\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D} \ = \ (P O_{\rm R} Q)^\dagger \, \overline{M}_{\rm D}^2 \, (P O_{\rm R} Q) \ , \tag{14}$$

where $P \equiv P_{\rm D}^T P_{\rm R}^* = {\rm Diag}\{1, e^{i\sigma}, 1\}$ with $\sigma \equiv \phi_{\rm D} - \phi_{\rm R}$. In obtaining this result, we have used the phase relation $\phi_{\rm D} - \varphi_{\rm D} = \phi_{\rm R} - \varphi_{\rm R}$. It is remarkable that only a single phase parameter σ contributes to $\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D}$. If σ vanishes, there will be no CP violation in the lepton-number-violating decays of heavy Majorana neutrinos N_i. We emphasize that σ has no direct connection with the effect of leptonic CP violation in neutrino oscillations. The latter is actually associated with the phase differences α and β appearing in Eq. (12) [8]. Only in the special case that ϕ_l and φ_l are switched off (or fixed to certain values), it is possible to indirectly link σ to α and β. For example, $\phi_l = \varphi_l = 0$ leads to $\beta = \sigma + \phi_{\rm D}$ and $\alpha = \varphi_\nu - \beta$.

With the help of Eqs. (6) and (14), we obtain the explicit expressions of $(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{11}$, ${\rm Im}[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{12}]^2$ and ${\rm Im}[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{13}]^2$ as follows:

$$\begin{aligned}(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{11} = & \; A_{\rm D}^2 (O_{31}^{\rm R})^2 + B_{\rm D}^2 \left[(O_{21}^{\rm R})^2 + (O_{31}^{\rm R})^2\right] + C_{\rm D}^2 \left[(O_{11}^{\rm R})^2 + (O_{21}^{\rm R})^2\right] \\ & + 2 A_{\rm D} B_{\rm D} O_{21}^{\rm R} O_{31}^{\rm R} \cos\sigma + 2 B_{\rm D} C_{\rm D} O_{11}^{\rm R} O_{31}^{\rm R} \ , \end{aligned} \tag{15}$$

and

$$\begin{aligned}{\rm Im}[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{12}]^2 = & -2 A_{\rm D} B_{\rm D} \left(O_{22}^{\rm R} O_{31}^{\rm R} - O_{21}^{\rm R} O_{32}^{\rm R}\right) \left[A_{\rm D}^2 O_{31}^{\rm R} O_{32}^{\rm R} + B_{\rm D}^2 \left(O_{21}^{\rm R} O_{22}^{\rm R} + O_{31}^{\rm R} O_{32}^{\rm R}\right) \right. \\ & + C_{\rm D}^2 \left(O_{11}^{\rm R} O_{12}^{\rm R} + O_{21}^{\rm R} O_{22}^{\rm R}\right) + A_{\rm D} B_{\rm D} \left(O_{21}^{\rm R} O_{32}^{\rm R} + O_{22}^{\rm R} O_{31}^{\rm R}\right) \cos\sigma \\ & \left. + B_{\rm D} C_{\rm D} \left(O_{11}^{\rm R} O_{32}^{\rm R} + O_{12}^{\rm R} O_{31}^{\rm R}\right)\right] \sin\sigma \ , \\ {\rm Im}[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{13}]^2 = & +2 A_{\rm D} B_{\rm D} \left(O_{23}^{\rm R} O_{31}^{\rm R} - O_{21}^{\rm R} O_{33}^{\rm R}\right) \left[A_{\rm D}^2 O_{31}^{\rm R} O_{33}^{\rm R} + B_{\rm D}^2 \left(O_{21}^{\rm R} O_{23}^{\rm R} + O_{31}^{\rm R} O_{33}^{\rm R}\right) \right. \\ & + C_{\rm D}^2 \left(O_{11}^{\rm R} O_{13}^{\rm R} + O_{21}^{\rm R} O_{23}^{\rm R}\right) + A_{\rm D} B_{\rm D} \left(O_{21}^{\rm R} O_{33}^{\rm R} + O_{23}^{\rm R} O_{31}^{\rm R}\right) \cos\sigma \\ & \left. + B_{\rm D} C_{\rm D} \left(O_{11}^{\rm R} O_{33}^{\rm R} + O_{13}^{\rm R} O_{31}^{\rm R}\right)\right] \sin\sigma \ . \end{aligned} \tag{16}$$

Combining Eqs. (13) and (16), one can clearly see that the CP-violating asymmetry ε_1 is proportional to $A_{\rm D}|B_{\rm D}|\sin\sigma$. Hence σ is the only source of CP violation for the decays of heavy right-handed neutrinos in our ansatz.

Apart from $\sin\beta_{\rm susy}$ or $\tan\beta_{\rm susy}$, the free parameters of ε_1 include m_i, M_i (for $i=1,2,3$) and σ. Current neutrino oscillation data allow us to constrain m_i or equivalently x_ν, z_ν and m_3 to an acceptable degree of accuracy [7]. The ratio

$$r \;\equiv\; \frac{B_{\rm R}}{C_{\rm R}} \;=\; \frac{B_\nu}{C_\nu} \;=\; \sqrt{\frac{(1-x_\nu)\,(1+z_\nu)\,(x_\nu-z_\nu)}{x_\nu z_\nu}} \tag{17}$$

can then be determined. Note that Eq. (17) remains valid, if (x_ν, z_ν) are replaced by $(x_{\rm R}, z_{\rm R})$. This implies that $x_{\rm R}$ and $z_{\rm R}$ are correlated with each other for a given value of r. Indeed, it is straightforward to obtain

$$z_{\rm R} = \frac{\sqrt{\left[(1-x_{\rm R})^2 + r^2 x_{\rm R}\right]^2 + 4x_{\rm R}\,(1-x_{\rm R})^2} - \left[(1-x_{\rm R})^2 + r^2 x_{\rm R}\right]}{2\,(1-x_{\rm R})} \;. \tag{18}$$

Taking account of Eq. (18), we are left with four unknown parameters to evaluate the magnitude of ε_1; namely, M_1, $x_{\rm R}$, σ and $\tan\beta_{\rm susy}$.

In the spirit of thermal leptogenesis [9], the CP-violating asymmetry ε_1 may lead to a net lepton number asymmetry $Y_{\rm L} \equiv (n_{\rm L} - n_{\bar{\rm L}})/\mathbf{s} = \varepsilon_1 d/g_*$, where $g_* = 228.75$ is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy $\mathbf{s}$ of the early universe, and d accounts for the dilution effects induced by the lepton-number-violating wash-out processes. This lepton number asymmetry is eventually converted into a net baryon number asymmetry $Y_{\rm B} \equiv (n_{\rm B} - n_{\bar{\rm B}})/\mathbf{s}$ via nonperturbative sphaleron processes [10] [1]: $Y_{\rm B} \approx -0.35 Y_{\rm L}$ in the MSSM with three fermion families and two Higgs doublets. Although the dilution factor d can be computed by solving the full set of Boltzmann equations, it is more convenient to adopt a simple analytical approximation of d proposed in Ref. [16]: $d \approx 0.02 \times (0.01 \text{ eV}/\tilde{m}_1)^{1.1}$, where $\tilde{m}_1 \equiv (\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{11}/M_1$ is an effective neutrino mass parameter and its plausible magnitude is expected to lie in the range 10^{-2} eV $\lesssim \tilde{m}_1 \lesssim 1$ eV (the so-called strong washout regime [16]). If our phenomenological ansatz of lepton mass matrices is viable for leptogenesis, it should be able to reproduce the observed magnitude of $Y_{\rm B}$ (i.e., $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$ [11]).

To evaluate ε_1 and $Y_{\rm B}$, we adopt $x_\nu \approx 1/3$, $z_\nu \approx 1/12$ and $m_3 \approx 0.05$ eV as favorable inputs given in Ref. [7]. In addition, we typically take $\tan\beta_{\rm susy} \approx 10$ and $x_{\rm R} \approx 1/4$. We allow M_1 and σ to vary, in order to reproduce $Y_{\rm B}$ in its afore-mentioned range. Then we arrive at the parameter space of M_1 and σ, as shown in Fig. 1. The lower bound of M_1 is about 2.5×10^{10} GeV, while the lower limit of σ approximately reads 8.0°. When M_1 is much higher than 10^{12} GeV, the value of σ becomes closer to 57°. Note that there is in

[1]We are grateful to Yanagida for clarifying our misunderstanding of Ref. [14] and calling our particular attention to Hamaguchi's PhD thesis [15], in which the relationship between $Y_{\rm B}$(final) and $Y_{\rm L}$(initial) is clearly discussed.

general a potential conflict between achieving successful thermal leptogenesis and avoiding overproduction of gravitinos in the conventional seesaw model with supersymmetry [17]. If the mass scale of gravitinos is of $\mathcal{O}(1)$ TeV, one must have $M_1 \lesssim 10^8$ GeV. This limit is apparently disfavored in our ansatz. Such a problem could be circumvented in other supersymmetric breaking mediation scenarios (e.g., gauge mediation [18]) or in a class of supersymmetric axion models [19], where the gravitino mass can be much lighter in spite of the very high reheating temperature.

It is worth pointing out that our ansatz can clearly be distinguished from the interesting Fukugita-Tanimoto-Yanagida (FTY) ansatz of lepton mass matrices [20] by comparing between their consequences on leptogenesis. In the FTY ansatz, M_l and $M_{\rm D}$ are also of the Fritzsch texture, but $M_{\rm R} = M_0 \mathbf{1}$ with $\mathbf{1}$ being the unit matrix has been assumed. Hence the texture of M_ν is different from that of M_l or $M_{\rm D}$. The exact mass degeneracy of three right-handed neutrinos (i.e.,$M_i = M_0$ for $i = 1, 2, 3$) immediately implies that the rephasing invariant of CP violation

$$\begin{aligned}\Delta_{\rm CP} &\equiv {\rm Im Tr}\left[Y_\nu^\dagger Y_\nu M_{\rm R}^\dagger M_{\rm R} M_{\rm R}^\dagger Y_\nu^T Y_\nu^* M_{\rm R}\right] \\ &= \frac{1}{\langle H_2\rangle^4}\sum_{i<j}\left\{M_i M_j\left(M_j^2 - M_i^2\right){\rm Im}\left[(\tilde{M}_{\rm D}^\dagger \tilde{M}_{\rm D})_{ij}\right]^2\right\} \ , \end{aligned} \tag{19}$$

which is relevant for leptogenesis [21], is actually vanishing. We can therefore conclude that there will be no lepton number asymmetry between $N_1 \to l + H_2^{\rm c}$ and $N_1 \to l^{\rm c} + H_2$ decays in the FTY ansatz, although it *does* allow leptonic CP violation to show up in low-energy neutrino oscillations. In order to accommodate leptogenesis, a slight modification of the FTY hypothesis is necessary. The simplest way might be to assume that three heavy right-handed neutrinos have a near mass degeneracy [22], such that the cosmological baryon number asymmetry may arise from the resonantly-enhanced thermal leptogenesis [23].

IV. LEPTON FLAVOR VIOLATION

We proceed to discuss the consequence of our phenomenological scenario on lepton flavor violation in the framework of MSSM with three heavy right-handed neutrinos. We restrict ourselves to a very simple and conservative case where supersymmetry is broken in a hidden sector and the breaking is transmitted to the observable sector by a flavor blind mechanism, such as gravity [18]. Then all the soft breaking terms are diagonal at high energy scales, and the only source of lepton flavor violation in the charged lepton sector is the radiative correction to the soft terms through the neutrino Yukawa couplings. In other words, the low-energy lepton-flavor-violating processes $l_j \to l_i \gamma$ (for $i, j = e, \mu, \tau$ and $m_j > m_i$) are induced by the renormalization-group effects of the slepton mixing. The off-diagonal elements of the left-handed slepton mass matrix can be written as [24]

$$\left(M_{\tilde{L}}^2\right)_{ij} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2 v^2 \sin^2\beta_{\rm susy}}\left(\tilde{M}_{\rm D} L \tilde{M}_{\rm D}^\dagger\right)_{ij} \tag{20}$$

in the leading-logarithmic approximation, where m_0 and A_0 denote the universal scalar soft mass and the trilinear term at the GUT scale, respectively. Note that the diagonal matrix

L in Eq. (19) measures the difference between the scale of heavy Majorana neutrinos and that of GUT,

$$L \;=\; \begin{pmatrix} \ln \frac{M_{\rm GUT}}{M_1} & 0 & 0 \\ 0 & \ln \frac{M_{\rm GUT}}{M_2} & 0 \\ 0 & 0 & \ln \frac{M_{\rm GUT}}{M_3} \end{pmatrix} \; . \tag{21}$$

The branching ratios of $l_j \to l_i \gamma$ can approximately be given by

$${\rm Br}(l_j \to l_i \gamma) \approx \frac{\alpha^3}{G_{\rm F}^2} \cdot \frac{|(M_{\tilde{L}}^2)_{ij}|^2}{m_{\rm S}^8} \tan^2 \beta_{\rm susy} \tag{22}$$

with $m_{\rm S}^8 \approx 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$, where $M_{1/2}$ denotes the gaugino mass [25]. Once the texture of $\tilde{M}_{\rm D}$ is specified, it will be possible to get concrete predicitions for ${\rm Br}(l_j \to l_i \gamma)$.

Given the Fritzsch-like textures of M_a (for $a = l, \nu, {\rm D}, {\rm R}$), the quantity $\tilde{M}_{\rm D} L \tilde{M}_{\rm D}^\dagger$ can be rewritten as

$$\tilde{M}_{\rm D} L \tilde{M}_{\rm D}^\dagger \;=\; \left(Q O_l^T P^\prime \overline{M}_{\rm D} P O_{\rm R} \right) L \left(Q O_l^T P^\prime \overline{M}_{\rm D} P O_{\rm R} \right)^\dagger \tag{23}$$

in the chosen flavor basis (i.e., both M_l and $M_{\rm R}$ are diagonal, real and positive), where $P^\prime \equiv P_l^\dagger P_{\rm D} = {\rm Diag}\{e^{i\alpha}, e^{i\rho}, 1\}$ with $\alpha \equiv (\varphi_{\rm D} - \phi_{\rm D}) - (\varphi_l - \phi_l)$ and $\rho \equiv \phi_{\rm D} - \phi_l$, and all the other matrices have been given before. Note that the phase α defined here is just the one defined in Eq. (12) for the lepton flavor mixing matrix V, because the phase equation $\varphi_{\rm D} - \phi_{\rm D} = \varphi_\nu - \phi_\nu = \varphi_{\rm R} - \phi_{\rm R}$ *does* hold in our ansatz. Furthermore, it is easy to prove

$$\rho + \sigma = 2\phi_{\rm D} - \phi_{\rm R} - \phi_l = \phi_\nu - \phi_l = \beta \; , \tag{24}$$

where the seesaw phase relation $\phi_\nu = 2\phi_{\rm D} - \phi_{\rm R}$ has been used. Let us summarize the phase parameters appearing in three categories of phenomena at this point:

$$\begin{aligned} &\text{Neutrino oscillations}: \; \alpha \text{ and } \beta \; ; \\ &\text{Thermal leptogenesis}: \; \sigma \; ; \\ &\text{Lepton flavor violation}: \; \alpha \text{ and } \rho = \beta - \sigma \; . \end{aligned}$$

We see that V, $Y_{\rm B}$ and ${\rm Br}(l_j \to l_i \gamma)$ totally involve three free phases. Among them, α and β can be determined from the precise measurement of lepton flavor mixing and CP violation in neutrino oscillations. Current neutrino oscillation data *do* favor $\beta \approx \pi$, but they are unable to provide a narrow constraint on α [7]. If the value of α is fixed, nevertheless, one may examine the dependence of ${\rm Br}(l_j \to l_i \gamma)$ on σ with the help of $\rho \approx \pi - \sigma$.

To illustrate, we take $\alpha \approx 30^\circ$, $\beta \approx 180^\circ$ and $x_{\rm R} \approx 1/4$ in addition to $x_\nu \approx 1/3$, $z_\nu \approx 1/12$ and $m_3 \approx 0.05$ eV [7]. The supersymmetric parameters relevant to our calculation are typically chosen as $\tan \beta_{\rm susy} \approx 10$, $A_0 \approx 0$, $m_0 \approx 100$ GeV, $M_{1/2} \approx 300$ GeV and $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV. Then we are left with two free parameters M_1 and σ, just like the case of thermal leptogenesis. Allowing M_1 and σ to vary, we calculate ${\rm Br}(l_j \to l_i \gamma)$ by using Eqs. (19)–(23) and by including the leptogenesis constraint $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$. The experimental upper bounds of ${\rm Br}(l_j \to l_i \gamma)$ should certainly be taken into account [26]:

$$\begin{aligned}
\mathrm{Br}(\mu \to e\gamma) &< 1.2 \times 10^{-11} \ , \\
\mathrm{Br}(\tau \to e\gamma) &< 3.6 \times 10^{-7} \ , \\
\mathrm{Br}(\tau \to \mu\gamma) &< 3.1 \times 10^{-7} \ .
\end{aligned} \tag{25}$$

Our numerical results are shown in Figs. 2–4. Once can see that the predicted branching ratios of $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are all below their corresponding experimental upper limits. A generic feature of $\mathrm{Br}(l_j \to l_i\gamma)$ is that they increase with the phase parameter σ. In particular, $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$ is reachable for $\sigma \sim 56°$. Of course, the magnitudes of $\mathrm{Br}(l_j \to l_i\gamma)$ will change, if different values of the supersymmetric parameters are input. Instead of examining the full parameter space, here we have paid our main attention to the parameter correlation between leptogenesis and lepton-flavor-violating processes.

Let us remark that the gravity mediation scenario used in evaluating lepton flavor violation is in potential conflict with the lower bound of M_1 obtained from thermal leptogenesis in our ansatz. This problem, usually referred to as the gravitino problem, exists in many supersymmetric seesaw models (see, e.g., Ref. [17] and references therein). One possible way out is to fine-tune the model parameters (i.e., those of Y_ν and $M_{\rm R}$) or assume the masses of heavy right-handed neutrinos to be nearly degenerate [23] within the thermal leptogenesis mechanism. As for our universal Fritzsch-like textures of lepton mass matrices, however, we are not left with much room for the fine-tuning of model parameters. The assumption of a near mass degeneracy for heavy Majorana neutrinos seems not to be natural either, although it is not impossible.

We find that there actually exists an interesting solution to the gravitino problem [19], which is in no apparent conflict with our phenomenological scenario. Its key point is that the axino and gravitino can be the lightest (of $\mathcal{O}(1)$ keV) and the next lightest (of $\mathcal{O}(10^2)$ GeV) supersymmetric particles, respectively, in a large class of supersymmetric axion models. Thus the reheating temperature is allowed to reach 10^{15} GeV or so [19], high enough for the thermal leptogenesis mechanism to work. Some more discussions in this connection will be presented elsewhere [22], since they are already beyond the scope of the present work.

V. SUMMARY

We have incorporated the seesaw mechanism with six Fritzsch-like textures of lepton mass matrices. It is found that the seesaw relation holds under a particular condition, and the consequences of those textures on lepton flavor mixing are exactly the same. Applying this simple ansatz to thermal leptogenesis, we have shown that CP violation in the lepton-number-violating decays of heavy right-handed neutrinos depends only upon a single phase parameter and the cosmological baryon number asymmetry can naturally be explained. The lepton-flavor-violating processes $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ have also been calculated. An interesting result is that the branching ratios of those rare processes rely strongly upon the phase parameters appearing in leptogenesis and in neutrino oscillations. We expect that a stringent test of our phenomenological scenario will be available in the near future, when more precise experimental data are accumulated.

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REFERENCES

[1] For a review, see: C.K. Jung *et al.*, Ann. Rev. Nucl. Part. Sci. **51**, 451 (2001).

[2] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).

[3] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).

[4] K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).

[5] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[6] See, e.g., H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); F. Feruglio, hep-ph/0410131; and references therein.

[7] Z.Z. Xing and S. Zhou, Phys. Lett. B **593**, 156 (2004); S. Zhou and Z.Z. Xing, hep-ph/0404188.

[8] Z.Z. Xing, Phys. Lett. B **550**, 178 (2002).

[9] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[10] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).

[11] D.N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).

[12] H. Fritzsch, Phys. Lett. B **73**, 317 (1978); Nucl. Phys. B **155**, 189 (1979).

[13] See, e.g., Z.Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004); and references therein.

[14] J.A. Harvey and M.S. Turner, Phys. Rev. D **42**, 3344 (1990).

[15] K. Hamaguchi, *PhD thesis*, hep-ph/0212305.

[16] W. Buchmüller, P. Di Bari, and M. Plümacher, hep-ph/0401240; hep-ph/0406014. A slightly different analytical approximation for d has been presented by Giudice *et al* in Ref. [17], but the numerical discrepancy between two approximations is insignificant. For an explicit comparison, see: Z.Z. Xing, hep-ph/0406047.

[17] G.F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, Nucl. Phys. B **685**, 89 (2004).

[18] See, e.g., A. Ibarra and G.G. Ross, Phys. Lett. B **591**, 285 (2004); and references therein.

[19] T. Asaka and T. Yanagida, Phys. Lett. B **494**, 297 (2000); and references therein.

[20] M. Fukugita, M. Tanimoto, and T. Yanagida, Prog. Theor. Phys. **89**, 263 (1993); Phys. Lett. B **562**, 273 (2003).

[21] A. Pilaftsis, Phys. Rev. D **56**, 5431 (1997); G.C. Branco, T. Morozumi, B.M. Nobre, and M.N. Rebelo, Nucl. Phys. B **617**, 475 (2001); G.C. Branco and M.N. Rebelo, hep-ph/0411196; and references therein.

[22] Z.Z. Xing and S. Zhou, in preparation.

[23] A. Pilaftsis and T.E.J. Underwood, Nucl. Phys. B **692**, 303 (2004).

[24] J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, and T. Yanagida, Phys. Lett. B **357**, 579 (1995); Phys. Rev. D **53**, 2442 (1996).

[25] S.T. Petcov, S. Profumo, Y. Takanishi, and C.E. Yaguna, Nucl. Phys. B **676**, 453 (2004); M. Bando, S. Kaneko, M. Obara, and M. Tanimoto, hep-ph/0405071.

[26] M. Aoki, talk given at *Neutrino 2004*, June 2004, Paris; and references therein.

TABLES

TABLE I. The Fritzsch-like lepton mass matrix M_a (for $a = l, \mathrm{D}, \mathrm{R}, \nu$), where $\arg(\mathcal{A}_a) \equiv 0$, $\arg(\mathcal{B}_a) \equiv \phi_a$ and $\arg(\mathcal{C}_a) \equiv \varphi_a$. The phase matrix P_a and the real orthogonal matrix O_a are defined to diagonalize M_a via the transformation $(P_a^* O_a Q)^T M_a (P_a^* O_a Q)$ with $Q \equiv \mathrm{Diag}\{1, i, 1\}$. The explicit expressions of O_{ij}^a (for $i, j = 1, 2, 3$) are given in Eq. (9).

Pattern	M_a	P_a	O_a
(A)	$\begin{pmatrix} \mathbf{0} & \mathcal{C}_a & \mathbf{0} \\ \mathcal{C}_a & \mathbf{0} & \mathcal{B}_a \\ \mathbf{0} & \mathcal{B}_a & \mathcal{A}_a \end{pmatrix}$	$\begin{pmatrix} e^{i(\varphi_a - \phi_a)} & 0 & 0 \\ 0 & e^{i\phi_a} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} O_{11}^a & O_{12}^a & O_{13}^a \\ O_{21}^a & O_{22}^a & O_{23}^a \\ O_{31}^a & O_{32}^a & O_{33}^a \end{pmatrix}$
(B)	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathcal{C}_a \\ \mathbf{0} & \mathcal{A}_a & \mathcal{B}_a \\ \mathcal{C}_a & \mathcal{B}_a & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} e^{i(\varphi_a - \phi_a)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_a} \end{pmatrix}$	$\begin{pmatrix} O_{11}^a & O_{12}^a & O_{13}^a \\ O_{31}^a & O_{32}^a & O_{33}^a \\ O_{21}^a & O_{22}^a & O_{23}^a \end{pmatrix}$
(C)	$\begin{pmatrix} \mathbf{0} & \mathcal{C}_a & \mathcal{B}_a \\ \mathcal{C}_a & \mathbf{0} & \mathbf{0} \\ \mathcal{B}_a & \mathbf{0} & \mathcal{A}_a \end{pmatrix}$	$\begin{pmatrix} e^{i\phi_a} & 0 & 0 \\ 0 & e^{i(\varphi_a - \phi_a)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} O_{21}^a & O_{22}^a & O_{23}^a \\ O_{11}^a & O_{12}^a & O_{13}^a \\ O_{31}^a & O_{32}^a & O_{33}^a \end{pmatrix}$
(D)	$\begin{pmatrix} \mathbf{0} & \mathcal{B}_a & \mathcal{C}_a \\ \mathcal{B}_a & \mathcal{A}_a & \mathbf{0} \\ \mathcal{C}_a & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} e^{i\phi_a} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\varphi_a - \phi_a)} \end{pmatrix}$	$\begin{pmatrix} O_{21}^a & O_{22}^a & O_{23}^a \\ O_{31}^a & O_{32}^a & O_{33}^a \\ O_{11}^a & O_{12}^a & O_{13}^a \end{pmatrix}$
(E)	$\begin{pmatrix} \mathcal{A}_a & \mathbf{0} & \mathcal{B}_a \\ \mathbf{0} & \mathbf{0} & \mathcal{C}_a \\ \mathcal{B}_a & \mathcal{C}_a & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\varphi_a - \phi_a)} & 0 \\ 0 & 0 & e^{i\phi_a} \end{pmatrix}$	$\begin{pmatrix} O_{31}^a & O_{32}^a & O_{33}^a \\ O_{11}^a & O_{12}^a & O_{13}^a \\ O_{21}^a & O_{22}^a & O_{23}^a \end{pmatrix}$
(F)	$\begin{pmatrix} \mathcal{A}_a & \mathcal{B}_a & \mathbf{0} \\ \mathcal{B}_a & \mathbf{0} & \mathcal{C}_a \\ \mathbf{0} & \mathcal{C}_a & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_a} & 0 \\ 0 & 0 & e^{i(\varphi_a - \phi_a)} \end{pmatrix}$	$\begin{pmatrix} O_{31}^a & O_{32}^a & O_{33}^a \\ O_{21}^a & O_{22}^a & O_{23}^a \\ O_{11}^a & O_{12}^a & O_{13}^a \end{pmatrix}$

FIGURES

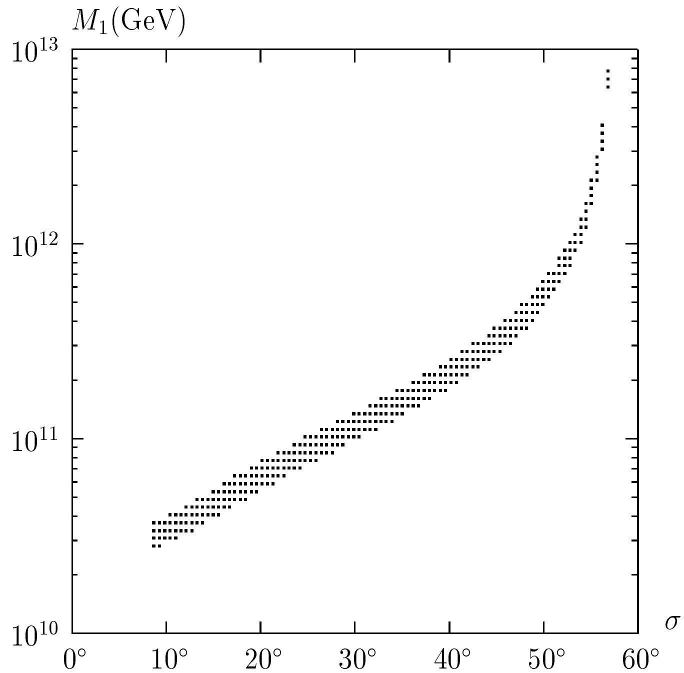

FIG. 1. The allowed ranges of σ and M_1 to reproduce $7 \times 10^{-11} \leq Y_{\rm B} \leq 10^{-10}$ via leptogenesis, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_{\rm R} \approx 1/4$ and $\tan\beta_{\rm susy} \approx 10$ have typically been input.

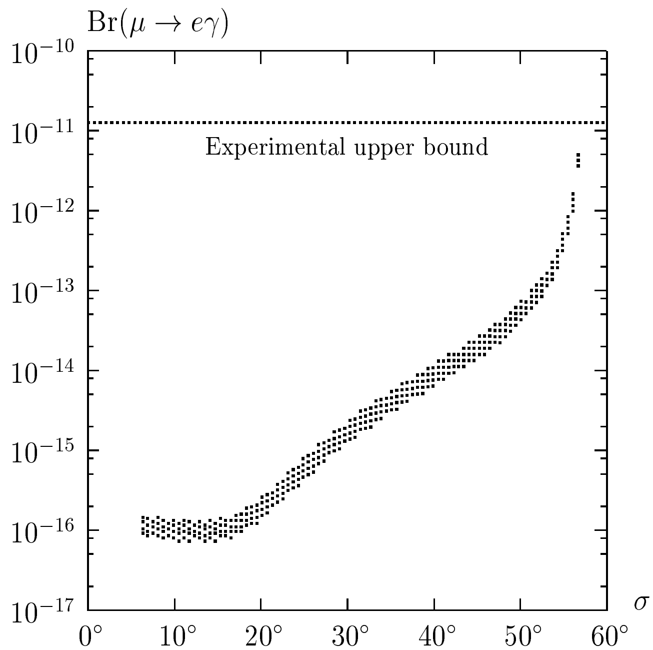

FIG. 2. The allowed ranges of σ and $\mathrm{Br}(\mu \to e\gamma)$, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_{\mathrm{R}} \approx 1/4$ and $\tan\beta_{\mathrm{susy}} \approx 10$ have typically been input.

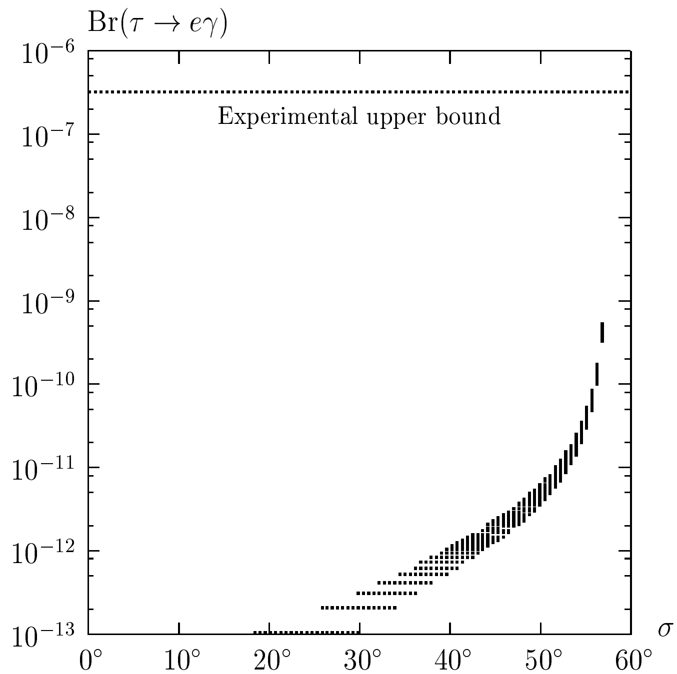

FIG. 3. The allowed ranges of σ and $\mathrm{Br}(\tau \to e\gamma)$, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_{\rm R} \approx 1/4$ and $\tan\beta_{\rm susy} \approx 10$ have typically been input.

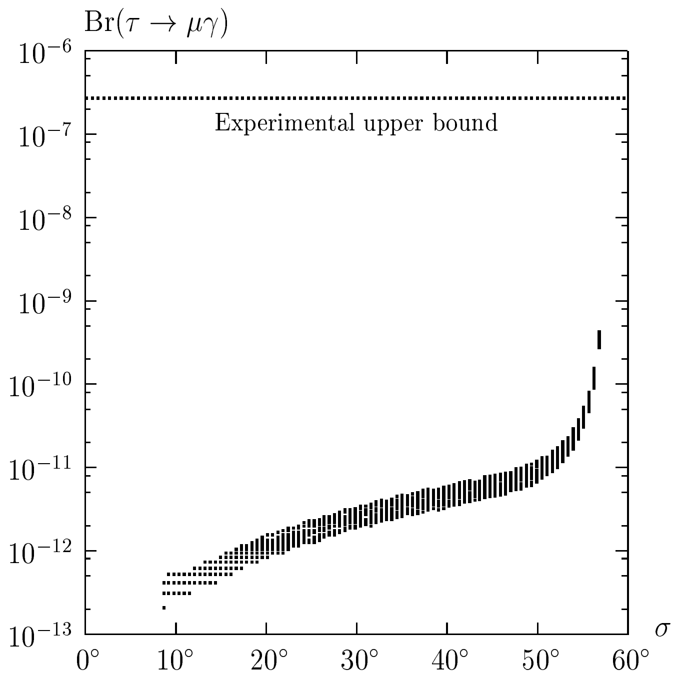

FIG. 4. The allowed ranges of σ and $\mathrm{Br}(\tau \to \mu\gamma)$, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_{\mathrm{R}} \approx 1/4$ and $\tan\beta_{\mathrm{susy}} \approx 10$ have typically been input.